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Research Article

Laplace Transform for the Solution of Non-Linear Volterra Integral Equation of Second Kind

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I N F O

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A B S T R A C T

The goal of this work is to solve the second-kind nonlinear Volterra integral problem. The compact form solution of the nonlinear Volterra integral problem of second kind was determined using the Laplace transform. Two numerical problems were investigated, and their solutions were determined in detail using the Laplace transform. The findings of this investigation indicate that the Laplace transform satisfactorily addressed the study's difficulty. By tackling the complexity of Volterra integral equations of the second kind, including their nonlinearity, the Laplace transform opens the way to solving real-world issues in a variety of disciplines.

Keywords: Laplace Transform, Convolution, Volterra Integral Equation, Dirac Delta Function

Introduction

Volterra integral equations are employed in many academic fields, including physics, biology, mechanics, and medicine, to explain a wide range of real-world issues.[1-3] Volterra integral equations can be solved numerically[4–9] and analytically.[10-26] The kind of the Volterra integral equations determines which of these approaches to use. Researchers used analytical methods to solve the linear Volterra integral equations, whether they were homogeneous or non-homogeneous. However, researchers solved the Volterra integral equations using numerical techniques if they were non-linear (homogeneous or non-homogeneous). Due to the non-linearity of the unknown functions, solving non-linear Volterra integral equations becomes increasingly difficult. Using a variety of integral transformations, researchers[27–49] have recently resolved issues in mathematics, biology, physics, chemistry, medical science, and mechanics. Aggarwal et al.[50–55] found the compact form solutions of the system of ordinary differential equations by comparing several integral transforms. The duality relation of recently discovered integral transforms was demonstrated by researchers[56–63], and all of these findings are extensively reported. Using the Laplace transform, a novel method for dealing with this circumstance is presented in this study. By using this integral transform, we are able to convert the problem into an algebraic equation that can be solved with ease using conventional methods, and in doing so, we are able to obtain the necessary answer for our non-linear problem.

Nomenclature of Symbols

$\mathbb{L}$, Laplace transform operator;

$\mathbb{L}^{-1}$, inverse Laplace transform operator;

N, the set of natural numbers;

$\in$, belongs to;

!, the usual factorial notation;

Γ, the classical Gamma function;

R, the set of real numbers

Definition of Laplace Transform

A sectionally continuous exponential order function $\eta(t), t \geq 0$, has the Laplace transform given by[31]

$$\mathbb{L}\{\eta(t)\} = \int_0^\infty \eta(t)e^{-\varepsilon t}dt = \mathcal{T}(\varepsilon),\ \varepsilon > 0 \qquad (1)$$

Inverse Laplace Transform[50]

The inverse Laplace transform of $\mathcal{T}(\varepsilon)$, designated by $\mathbb{L}^{-1}\{\mathcal{T}(\varepsilon)\}$, is another function $\eta(t)$ having the property that $\mathbb{L}\{\eta(t)\} = \mathcal{T}(\varepsilon)$.

Tables 1-3 provide an overview of the Laplace transform's significant operational features, as well as the Laplace transforms of a few basic functions and their inverse Laplace transforms.

Table 1. Some significant operational features of Laplace transform[31]

S.N.	Name of Characteristic	Mathematical Form
1	Linearity	$\mathbb{L}\{\sum_{i=1}^{n} a_i\eta_i(t)\} = \sum_{i=1}^{n} a_i\,\mathbb{L}\{\eta_i(t)\}$, where a_i are arbitrary constants
2	Change of Scale	If $\mathbb{L}\{\eta(t)\} = \mathcal{T}(\varepsilon)$ then $\mathbb{L}\{\eta(at)\} = \frac{1}{a}\mathcal{T}\left(\frac{\varepsilon}{a}\right)$
3	Translation	If $\mathbb{L}\{\eta(t)\} = \mathcal{T}(\varepsilon)$ then $\{\mathbb{L}\{e^{at}\eta(t)\} = \mathcal{T}(\varepsilon - a)\}$
4	Convolution	If $\mathbb{L}\{\eta_1(t)\} = \mathcal{T}_1(\varepsilon)$ and $\mathbb{L}\{\eta_2(t)\} = \mathcal{T}_2(\varepsilon)$ then $\{\mathbb{L}\{\eta_1(t) * \eta_2(t)\} = \mathbb{L}\{\eta_1(t)\}\mathbb{L}\{\eta_2(t)\} = \mathcal{T}_1(\varepsilon)\,\mathcal{T}_2(\varepsilon)\}$

Table 2. Laplace transforms of a few basic functions[60]

S.N.	$\eta(t), t > 0$	$\mathbb{L}\{\eta(t)\} = \mathcal{T}(\varepsilon)$
1	1	$\left(\frac{1}{\varepsilon}\right)$
2	e^{at}	$\frac{1}{(\varepsilon - a)}$
3	$t^a, a \in N$	$a!\left(\frac{1}{\varepsilon}\right)^{a+1}$

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4	$t^a, a > -1, a \in R$	$\left(\frac{1}{\varepsilon}\right)^{a+1} \Gamma(a+1)$
5	$sin(at)$	$\frac{a}{(\varepsilon^2 + a^2)}$
6	$cos(at)$	$\frac{\varepsilon}{(\varepsilon^2 + a^2)}$
7	$sinh(at)$	$\frac{a}{(\varepsilon^2 - a^2)}$
8	$cosh(at)$	$\frac{\varepsilon}{(\varepsilon^2 - a^2)}$

Table 3. Inverse Laplace transforms of a few basic functions[11]

S.N.	$\mathcal{T}(\varepsilon)$	$\eta(t) = \mathbb{L}^{-1}\{\mathcal{T}(\varepsilon)\}$
1	$\left(\frac{1}{\varepsilon}\right)$	1
2	$\frac{1}{(\varepsilon - a)}$	e^{at}
3	$\left(\frac{1}{\varepsilon}\right)^{a+1}, a \in N$	$\frac{t^a}{a!}$
4	$\left(\frac{1}{\varepsilon}\right)^{a+1}, a > -1, a \in R$	$\frac{t^a}{\Gamma(a+1)}$
5	$\frac{1}{(\varepsilon^2 + a^2)}$	$\frac{sin(at)}{a}$
6	$\frac{\varepsilon}{(\varepsilon^2 + a^2)}$	$cos(at)$
7	$\frac{1}{(\varepsilon^2 - a^2)}$	$\frac{sinh(at)}{a}$
8	$\frac{\varepsilon}{(\varepsilon^2 - a^2)}$	$cosh(at)$

Mean Value Theorem for Integrals[64]

If a function $\eta(t)$ is continuous on $[k_1, k_2]$, then there exists a number u in $[k_1, k_2]$ such that

$$\int_{k_1}^{k_2} \eta(t)\, dt = \eta(u)(k_2 - k_1) \tag{2}$$

Dirac Delta Function[65]

The Dirac delta function is considered as the limiting form of the function

$$\delta_\omega(t - \ell) = \begin{cases} \frac{1}{\omega}, \ell \le t \le \ell + \omega \\ 0, otherwise \end{cases}$$

as, $\omega \to 0$.

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Laplace Transform of Dirac Delta Function[65]

If $\eta(t)$ is a continuous function at $t = \ell$, then

$\int_0^\infty \eta(t)\,\delta_\omega(t-\ell)dt = \int_\ell^{\ell+\omega} \eta(t)\frac{1}{\omega}dt = \eta(u)(\ell+\omega-\ell)\frac{1}{\omega} = \eta(u),\ \ell < u < l+\omega$ by equation (2).

As $\omega \to 0$, we have $\int_0^\infty \eta(t)\,\delta(t-\ell)dt = \eta(\ell)$.

In particular, when $\eta(t) = e^{-\varepsilon t}$, we have

$$\int_0^\infty e^{-\varepsilon t}\,\delta(t-\ell)dt = e^{-\varepsilon \ell}$$

$\Rightarrow \mathbb{L}\{\delta(t-\ell)\} = e^{-\varepsilon\ell}$

$\Rightarrow \mathbb{L}\{\delta(t)\} = 1.$

Laplace Transform for the Solution of Non-Linear Volterra Integral Equation of Second Kind

We assumed the following form of nonlinear Volterra integral equation of second kind in this paper.

$$\mathcal{G}(t) = \eta(t) + \int_0^t \mathcal{G}(t-x)\,\mathcal{G}(x)dx\ , \tag{3}$$

where $\mathcal{G}(t)$ and $\eta(t)$ are unknown and known functions respectively.

Operating Laplace transform on equation (3), we get

$$\mathbb{L}\{\mathcal{G}(t)\} = \mathbb{L}\{\eta(t)\} + \mathbb{L}\left\{\int_0^t \mathcal{G}(t-x)\,\mathcal{G}(x)dx\right\}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \mathbb{L}\{\eta(t)\} + \mathbb{L}\{\mathcal{G}(t) * \mathcal{G}(t)\} \tag{4}$$

Use of convolution theorem in equation (4) gives

$$\mathbb{L}\{\mathcal{G}(t)\} = \mathbb{L}\{\eta(t)\} + \mathbb{L}\{\mathcal{G}(t)\}\mathbb{L}\{\mathcal{G}(t)\}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \mathbb{L}\{\eta(t)\} + [\mathbb{L}\{\mathcal{G}(t)\}]^2$$

$$\Rightarrow [\mathbb{L}\{\mathcal{G}(t)\}]^2 - \mathbb{L}\{\mathcal{G}(t)\} + \mathbb{L}\{\eta(t)\} = 0$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \frac{1 \pm \sqrt{1 - 4\mathbb{L}\{\eta(t)\}}}{2}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \frac{1}{2}\left[1 \pm \left(\sqrt{1 - 4\mathbb{L}\{\eta(t)\}}\right)\right] \tag{5}$$

After operating inverse Laplace transform on equation (5), the solutions of equation (3) are given by

$$\mathcal{G}(t) = \mathbb{L}^{-1}\left\{\frac{1}{2}\left[1 \pm \left(\sqrt{1 - 4\mathbb{L}\{\eta(t)\}}\right)\right]\right\} \tag{6}$$

Numerical Problems: Two numerical problems are resented in this part to help Illustrate the entire process of finding the precise solution of the non-linear Volterra integral equation of second kind.

Problem: 8.1 Take into account the following nonlinear second-kind Volterra integral equation

$$\mathcal{G}(t) = 1 - \frac{t}{2} + \frac{1}{2}\int_0^t \mathcal{G}(t-x)\,\mathcal{G}(x)dx \tag{7}$$

Solution: Operating Laplace transform on equation (7), we get

$$\mathbb{L}\{\mathcal{G}(t)\} = \mathbb{L}\{1\} - \frac{1}{2}\mathbb{L}\{t\} + \frac{1}{2}\mathbb{L}\left\{\int_0^t \mathcal{G}(t-x)\,\mathcal{G}(x)dx\right\}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \left(\frac{1}{\varepsilon}\right) - \frac{1}{2}\left(\frac{1}{\varepsilon}\right)^2 + \frac{1}{2}\mathbb{L}\{\mathcal{G}(t) * \mathcal{G}(t)\} \tag{8}$$

Using convolution theorem in equation (8), we have

$$\mathbb{L}\{\mathcal{G}(t)\} = \left(\frac{1}{\varepsilon}\right) - \frac{1}{2}\left(\frac{1}{\varepsilon}\right)^2 + \frac{1}{2}\mathbb{L}\{\mathcal{G}(t)\}\mathbb{L}\{\mathcal{G}(t)\}$$

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$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \left(\frac{1}{\varepsilon}\right) - \frac{1}{2}\left(\frac{1}{\varepsilon}\right)^2 + \frac{1}{2}[\mathbb{L}\{\mathcal{G}(t)\}]^2$$

$$\Rightarrow \frac{1}{2}[\mathbb{L}\{\mathcal{G}(t)\}]^2 - \mathbb{L}\{\mathcal{G}(t)\} + \left(\frac{1}{\varepsilon}\right) - \frac{1}{2}\left(\frac{1}{\varepsilon}\right)^2 = 0$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \frac{1 \pm \sqrt{1 - 4\left(\frac{1}{2}\right)\left[\left(\frac{1}{\varepsilon}\right) - \frac{1}{2}\left(\frac{1}{\varepsilon}\right)^2\right]}}{2\left(\frac{1}{2}\right)}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = 1 \pm \sqrt{1 - 2\left[\left(\frac{1}{\varepsilon}\right) - \frac{1}{2}\left(\frac{1}{\varepsilon}\right)^2\right]}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = 1 \pm \sqrt{1 - 2\left(\frac{1}{\varepsilon}\right) + \left(\frac{1}{\varepsilon}\right)^2}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = 1 \pm \left[1 - \left(\frac{1}{\varepsilon}\right)\right]$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = 2 - \left(\frac{1}{\varepsilon}\right) \text{ and } \left(\frac{1}{\varepsilon}\right) \tag{9}$$

After operating inverse Laplace transform on equation (9), the solutions of equation (7) are given by

$$\mathcal{G}(t) = \mathbb{L}^{-1}\left\{2 - \left(\frac{1}{\varepsilon}\right)\right\} \text{ and } \mathbb{L}^{-1}\left\{\left(\frac{1}{\varepsilon}\right)\right\}$$

$$\Rightarrow \mathcal{G}(t) = 2\mathbb{L}^{-1}\{1\} - \mathbb{L}^{-1}\left\{\left(\frac{1}{\varepsilon}\right)\right\} \text{ and } \mathbb{L}^{-1}\left\{\left(\frac{1}{\varepsilon}\right)\right\}$$

$$\Rightarrow \mathcal{G}(t) = [2\delta(t) - 1] \text{ and } 1.$$

Problem: 8.2 Take into account the following nonlinear second-kind Volterra integral equation

$$\mathcal{G}(t) = (4t + 2)e^t - \int_0^t \mathcal{G}(t - x)\,\mathcal{G}(x)dx \tag{10}$$

Solution: Operating Laplace transform on equation (10), we get

$$\mathbb{L}\{\mathcal{G}(t)\} = 4\mathbb{L}\{te^t\} + 2\mathbb{L}\{e^t\} - \mathbb{L}\left\{\int_0^t \mathcal{G}(t - x)\,\mathcal{G}(x)dx\right\}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = 4\left[\frac{\sigma^3}{\varepsilon(\varepsilon-\sigma)^2}\right] + 2\left[\frac{1}{(\varepsilon-1)}\right] - \mathbb{L}\{\mathcal{G}(t) * \mathcal{G}(t)\} \tag{11}$$

Using convolution theorem in equation (11), we have

$$\mathbb{L}\{\mathcal{G}(t)\} = 4\left[\frac{1}{(\varepsilon-1)^2}\right] + 2\left[\frac{1}{(\varepsilon-1)}\right] - \mathbb{L}\{\mathcal{G}(t)\}\mathbb{L}\{\mathcal{G}(t)\}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = 4\left[\frac{1}{(\varepsilon-1)^2}\right] + 2\left[\frac{1}{(\varepsilon-1)}\right] - [\mathbb{L}\{\mathcal{G}(t)\}]^2$$

$$\Rightarrow [\mathbb{L}\{\mathcal{G}(t)\}]^2 + \mathbb{L}\{\mathcal{G}(t)\} - 4\left[\frac{1}{(\varepsilon-1)^2}\right] - 2\left[\frac{1}{(\varepsilon-1)}\right] = 0$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \frac{-1 \pm \sqrt{1 - 4\left[-4\left\{\frac{1}{(\varepsilon-1)^2}\right\} - 2\left\{\frac{1}{(\varepsilon-1)}\right\}\right]}}{2}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \frac{-1 \pm \sqrt{1 - \left[-16\left\{\frac{1}{(\varepsilon-1)^2}\right\} - 8\left\{\frac{1}{(\varepsilon-1)}\right\}\right]}}{2}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \frac{-1 \pm \sqrt{1 + 16\left\{\frac{1}{(\varepsilon-1)^2}\right\} + 8\left\{\frac{1}{(\varepsilon-1)}\right\}}}{2}$$

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$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = \frac{-1 \pm \left[1 + 4\left\{\frac{1}{(\varepsilon - 1)}\right\}\right]}{2}$$

$$\Rightarrow \mathbb{L}\{\mathcal{G}(t)\} = 2\left[\frac{1}{(\varepsilon-1)}\right] \text{ and } -1 - 2\left[\frac{1}{(\varepsilon-1)}\right] \quad (12)$$

After operating inverse Laplace transform on equation (12), the solutions of equation (10) are given by

$$\mathcal{G}(t) = 2\mathbb{L}^{-1}\left\{\frac{1}{(\varepsilon-1)}\right\} \text{ and } \mathbb{L}^{-1}\left\{-1 - 2\left[\frac{1}{(\varepsilon-1)}\right]\right\}$$

$$\Rightarrow \mathcal{G}(t) = 2\mathbb{L}^{-1}\left\{\frac{1}{(\varepsilon-1)}\right\} \text{ and } \left[-\mathbb{L}^{-1}\{1\} - 2\mathbb{L}^{-1}\left\{\frac{1}{(\varepsilon-1)}\right\}\right]$$

$$\Rightarrow \mathcal{G}(t) = 2e^{t} \text{ and } [-\delta(t) - 2e^{t}].$$

Conclusion

In the work presented, the authors successfully solved the non-linear Volterra integral equation of second kind in compact form by utilizing the LaPlace transform. The research findings indicate that the Laplace transform is a very useful integral transform for solving a non-linear Volterra integral equation of the second kind in compact form without requiring a significant amount of laborious computing work. In the future, complex scientific and engineering problems that can be reduced to one or more non-linear Volterra integral equations of the second class might be resolved using the Laplace transform.

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